فهرست مطالب

فصل ۱

روش‌های تقریبی و عددی

گامهای بلندی که اخیراً درنظریه معادلات دیفرانسیل جزیی غیرخطی برداشته شده است به اندازه گام‌های موجود در نظریه خطی است. برخلاف حالت خطی ، هیچ تسویه عمده از دسته وسیع از مسئله‌ها رخ نداده است، درعوض، پیشرفت روی مرزهای قدیمی و مرزهای جدید، حل کامل برخی ازمسئله‌های خاص و کشف برخی از پدیده‌های کاملاً تازه یکنواخت است . ابزار قدیمی-روش‌های تغییراتی، قضیه‌های نقطه ثابت ،درجه نگاشت و برخی ابزار توپولوژیکی دیگر به برخی ازابزار قدیمی اضافه شده اند. پیشرو در کشف پدیده جدید آزمایش عددی است؛ البته به احتمال، محاسبات عددی در آینده قسمت‌هایی از اثبات خواهند بود.

پیتر لکس [1]

قریب به یقین هرکسی که رایانه استفاده می کند نمونه‌هایی را دیده است که نتایج عددی آن منجر به بینش جدید شده است.

نرمن جی. زابوسکی [2]

۱.۱ مقدمه

در فصل‌های قبل به حل تحلیلی معادلات دیفرانسیل با مشتقات جزئی خطی و غیرخطی پرداختیم و چندین روش تحلیلی را برای یافتن جواب تحلیلی این معادلات در یک دامنه ساده مورد بحث قرار دادیم. شرایط مرزی و اولیه در این مسائل نسبتاً ساده و به صورت‌های ساده ریاضی قابل بیان بودند. در برخورد با مسائلی که از مدل‌بندی مسائل فیزیکی بوجود می‌آیند، تعیین جواب دقیق در یک دامنه ساده حتی با شرایط اولیه و مرزی ساده بسیار دشوار خواهد بود. پس متوسل شدن به روش‌های عددی یا تقریبی برای حل مسائلی که به‌صورت تحلیلی قابل حل نمی‌باشند، ضروری می‌باشد. با توجه به دسترسی گسترده به رایانه‌های بسیار سریع امروزی روش‌های عددی در کاربرد مهمتر و سودمندتر می‌شوند.

در این فصل برخی از روش‌های عددی و تقریبی مهم برای حل معادلات با مشتقات جزئی را مورد بحث قرار خواهیم داد. روش‌های عددی مورد بحث شامل: تقریبات تفاضل متناهی، روش‌های تغییراتی، ریلی-ریتز، گالرکین و روش کانتوروویچ می‌باشند. همچنین بخش اعظمی از این فصل مربوط به تحلیل روش‌های تغییراتی و معادلات اویلر-لاگرانژ و کاربردهای آن‌ها می‌باشد. در نهایت بخش کوتاهی در مورد روش عنصر متناهی خواهیم داشت.

۲.۱ تقریبات تفاضلی متناهی، همگرایی و پایداری

بسط سری تیلور تابع $u(x,y)$ از دو متغیر مستقل x و y به‌صورت

$$u(x_i \pm h, y_j) = u_{i\pm ۱,j} = u_{i,j} \pm h(u_x)_{i,j} + \frac{h^۲}{۲!}(u_{xx})_{i,j} \pm \frac{h^۳}{۳!}(u_{xxx})_{i,j} + ..., \qquad (۱.۲.۱)$$

$$u(x_i, y_j \pm k) = u_{i,j\pm ۱} = u_{i,j} \pm k(u_y)_{i,j} \pm \frac{k^۲}{۲!}(u_{yy})_{i,j} \pm \frac{k^۳}{۳!}(u_{yyy})_{i,j} + ..., \qquad (۲.۲.۱)$$

می‌باشد که در آن

$$u_{i,j} = u(x,y),\ u_{i\pm ۱,j} = u(x\pm h, y),\ u_{i,j\pm ۱} = u(x, y\pm k).$$

[1] Peter Lax
[2] Norman J. Zabusky

فصل ۲

جداول تبدیلات انتگرال

در این فصل جدولی از تبدیلات انتگرال توابعی که در کتاب آمده و یا توابعی که در ریاضی، فیزیک و مهندسی کاربرد فراوانی دارند آورده شده است. برای دسترسی به لیست کاملی از این تبدیلات خواننده می‌تواند به [۵٤، ۱۷٦، ٤۸، ٤۹، ۱۸۲، ۳۵، ۱۸۱] مراجعه نماید.

۱.۲ تبدیلات فوریه

$f(x)$	$F(k)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp(-ikx)f(x)dx$
$\exp(-\mathrm{a}\lvert\mathrm{x}\rvert),\quad a>0$	$\left(\sqrt{\frac{2}{\pi}}\right)a(a^2+k^2)^{-1}$
$x\exp(-a\lvert x\rvert),\quad a>0$	$\left(\sqrt{\frac{2}{\pi}}\right)(-2aik)(a^2+k^2)^{-2}$
$\exp(-ax^2),\quad a>0$	$\frac{1}{\sqrt{2a}}\exp\left(-\frac{k^2}{4a}\right)$
$(x^2+a^2)^{-1},\quad a>0$	$\sqrt{\frac{\pi}{2}}\frac{\exp(-\mathrm{a}\lvert\mathrm{k}\rvert)}{a}$
$x(x^2+a^2)^{-1}$	$\sqrt{\frac{\pi}{2}}\left(\frac{ik}{2a}\right)\exp(-a\lvert k\rvert)$
$\exp(iax)$	$\sqrt{2\pi}\delta(k-a)$

پیوست آ

دیوید هیلبرت [1]

این پیوست مقدمه‌ای کوتاه برای برخی از توابع خاص استفاده شده در کتاب می‌باشد. این توابع عبارتند از گاما، بتا، خطا و آیری و خواص اصلی آن‌ها. همچنین این پیوست توابع هرمیت، وبر–هرمیت و خواص آن‌ها را شامل می‌شود. از آن‌جایی که فرض براین است خواننده با این توابع آشنایی دارد، بحث مختصر خواهد بود. برای جزئیات بیشتر خوانند می‌تواند به کتاب‌های اشاره شده در منابع مراجعه نماید.

آ.۱ توابع گاما، بتا، خطا و آیری

تابع گاما (همچنین تابع فاکتوریل نیز نامیده می‌شود) به‌صورت انتگرال معین

$$\Gamma(x) = \int_{0}^{\infty} e^{-t} t^{x-1} dt, \quad x > 0 \tag{آ.۱.۱}$$

تعریف می‌شود. با توجه به این حقیقت که انتگرال (آ.۱.۱) برای هر x در بازه $[a, b]$ که $0 < a \leq b < \infty$، همگرای یکنواخت می‌باشد، $\Gamma(x)$ برای هر $x > 0$ تابعی پیوسته می‌باشد.

با انتگرال‌گیری جزبه‌جز از (آ.۱.۱) خاصیت اساسی تابع $\Gamma(x)$ را به‌صورت

$$\begin{aligned}\Gamma(x) &= [-e^{-t} t^{x-1}]_{0}^{\infty} + (x-1) \int_{0}^{\infty} e^{-t} t^{x-2} dt \\ &= (x-1)\Gamma(x-1), \qquad x-1 > 0\end{aligned}$$

داریم. با جایگذاری $x+1$ به‌جای x در رابطه فوق نتیجه اساسی

$$\Gamma(x+1) = x\Gamma(x) \tag{آ.۱.۲}$$

[1] David Hilbert

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