

$$\begin{aligned}
& |Uy(t) - U\tilde{y}(t)| \leq L|y(t) - \tilde{y}(t)| \\
& + \int_0^{\gamma_n} \omega_1(t, s) \left[|y(s) - \tilde{y}(s)| + \int_0^s \bar{\omega}_1(t, s, r) |y(r) - \tilde{y}(r)| dr \right] ds \\
& + \int_{\gamma_n}^t \omega_1(t, s) \left[|y(s) - \tilde{y}(s)| + \int_0^{\gamma_n} \bar{\omega}_1(t, s, r) |y(r) - \tilde{y}(r)| dr \right. \\
& \quad \left. + \int_{\gamma_n}^s \bar{\omega}_1(t, s, r) |y(r) - \tilde{y}(r)| dr \right] ds \\
& \leq L|y(t) - \tilde{y}(t)| + \gamma_n(\tilde{\omega}_{1n} + \gamma_n \bar{\omega}_{1n})|y - \tilde{y}|_{\gamma_n} \\
& \quad + \int_{\gamma_n}^t \tilde{\omega}_{1n} [|y(s) - \tilde{y}(s)| + \gamma_n \bar{\omega}_{1n} |y - \tilde{y}|_{\gamma_n} \\
& \quad \quad + \bar{\omega}_{1n} \int_{\gamma_n}^s |y(r) - \tilde{y}(r)| dr] ds \\
& \leq L|y(t) - \tilde{y}(t)| + \gamma_n[\tilde{\omega}_{1n} + \gamma_n \bar{\omega}_{1n} + (n - \gamma_n)\tilde{\omega}_{1n}\bar{\omega}_{1n}]|y - \tilde{y}|_{\gamma_n} \\
& \quad + \tilde{\omega}_{1n}[1 + (n - \gamma_n \bar{\omega}_{1n}) \int_{\gamma_n}^t |y(s) - \tilde{y}(s)| ds].
\end{aligned}$$