

# Groupoid I

Example

## Groupoid II

Let  $X$  be a manifold and

$$PH(X) = \{(x, [\sigma], y) \mid x, y \in X\}$$

where  $[\sigma]$  is the homotopy class of paths such that  $\sigma(0) = x$ ,  $\sigma(1) = y$ . Then  $PH(X)$  is a groupoid on  $X$  with the rules:  
 $\alpha(x, [\sigma], y) = x$ ,  $\beta(x, [\sigma], y) = y$ ,  
 $\mu((x, [\sigma], y), (y', [\tau], z)) = (x, [\sigma \circ \tau], z)$  if and only if  $y = y'$ ,  
where  $\sigma \circ \tau$  is the concatenation of paths  $\sigma$  and  $\tau$ ,  
 $\epsilon(x) = (x, [\text{constant}], x)$  and  $i(x, [\sigma], y) = (y, [\sigma^{-1}], x)$ , where  
 $\sigma^{-1}(t) = \sigma(1 - t)$ ,  $t \in [0, 1]$ .  $(PH(X), \alpha, \beta, \epsilon, i, \mu; X)$  is a groupoid. If  $PH(X)$  is equipped with the quotient topology of the compact open topology on the space of paths of  $X$ , then  $\alpha \times \beta : PH(X) \rightarrow X \times X$  is a covering map. It follows that  $PH(X)$  is a Lie groupoid on  $X$ , called it the Poincaré groupoid associated to  $X$ . Then  $(PH(X), \alpha, \beta, \epsilon, i, \mu; X)$  is a Lie groupoid