Groupoid

Example

Let X *be a manifold and*

$$PH(X) = \{(x, [\sigma], y) \mid x, y \in X\}$$

where $[\sigma]$ *is the homotopy class of paths such that* $\sigma(0) = x$, $\sigma(1) = y$. *Then* $PH(X)$ *is a groupoid on* X *with the rules:*
$\alpha(x, [\sigma], y) = x$, $\beta(x, [\sigma], y) = y$,
$\mu((x, [\sigma], y), (y', [\tau], z)) = (x, [\sigma \circ \tau], z)$ *if and only if* $y = y'$,
where $\sigma \circ \tau$ *is the concatenation of paths* σ *and* τ,
$\epsilon(x) = (x, [constant], x)$ *and* $i(x, [\sigma], y) = (y, [\sigma^{-1}], x)$, *where* $\sigma^{-1}(t) = \sigma(1 - t)$, $t \in [0, 1]$. $(PH(X), \alpha, \beta, \epsilon, i, \mu; X)$ *is a groupoid.*

Groupoid

Example (Cont.)

If $PH(X)$ is equipped with the quotient topology of the compact open topology on the space of paths of X, then $\alpha \times \beta : PH(X) \to X \times X$ is a covering map. It follows that $PH(X)$ is a Lie groupoid on X, called it the Poincaré groupoid associated to X. Then $(PH(X), \alpha, \beta, \epsilon, i, \mu; X)$ is a Lie groupoid.