5.2 The Laplace transform

5.2.1 Definition and notation

We define the Laplace transform of a function $f(t)$ by the expression

$$\mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt \tag{5.1}$$

where s is a complex variable and e^{-st} is called the **kernel** of the transformation.

It is usual to represent the Laplace transform of a function by the corresponding capital letter, so that we write

$$\mathscr{L}\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,dt \tag{5.2}$$

An alternative notation in common use is to denote $\mathscr{L}\{f(t)\}$ by $\bar{f}(s)$ or simply $\bar{f}$.

Before proceeding, there are a few observations relating to the definition (5.2) worthy of comment.

(a) The symbol $\mathscr{L}$ denotes the **Laplace transform operator**; when it operates on a function $f(t)$, it transforms it into a function $F(s)$ of the complex variable s. We say the operator transforms the function $f(t)$ in the t domain (usually called the **time domain**) into the function $F(s)$ in the s domain (usually called the **complex frequency domain**, or simply the **frequency domain**). This relationship is depicted graphically in Figure 5.2, and it is usual to refer to $f(t)$ and $F(s)$ as a **Laplace transform pair**, written as $\{f(t), F(s)\}$.

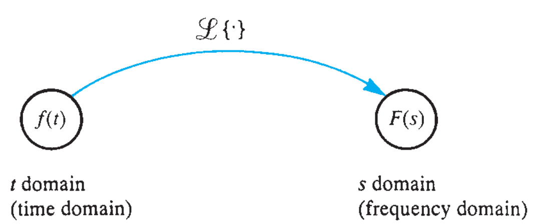

Figure 5.2 The Laplace transform operator.

(b) Because the upper limit in the integral is infinite, the domain of integration is infinite. Thus the integral is an example of an **improper integral**, as introduced in Section 9.2 of *Modern Engineering Mathematics*; that is,

$$\int_0^\infty e^{-st} f(t)\,dt = \lim_{T\to\infty} \int_0^T e^{-st} f(t)\,dt$$

This immediately raises the question of whether or not the integral converges, an issue we shall consider in Section 5.2.3.

(c) Because the lower limit in the integral is zero, it follows that when taking the Laplace transform, the behaviour of $f(t)$ for negative values of t is ignored or