

$$\Psi'(x)=\frac{\mp f(x)f'''(x)}{f''(x)\sqrt{f'^{\P}(x)-\P f(x)f''(x)}}+\frac{f'''(x)\Big[f'(x)\mp\sqrt{f'^{\P}(x)-\P f(x)f''(x)}\Big]}{f''^{\P}(x)}\tag{1}$$

$$\Psi'(x)=\frac{\mp f(x)f'''(x)}{f''(x)\sqrt{f'^{\P}(x)-\P f(x)f''(x)}}+\frac{f'''(x)\Big[f'(x)\mp\sqrt{f'^{\P}(x)-\P f(x)f''(x)}\Big]}{f''^{\P}(x)}\tag{2}$$

$$\begin{aligned} |p_nf(x_n)|+\P-q&\leq \frac{|p_nf^{\P}(x_n)+f(x_n)|-|f(q_n)|}{|f(x_n)|}\\ &\leq |\frac{p_nf^{\P}(x_n)+f(x_n)-f(q_n)}{f(x_n)}|\\ &\leq |p_nf(x_n)|+\P+q \end{aligned}$$